Digital downconversion stage design for superheterodyne receivers

Fast and accurate DSP technologies bring digital functionality to analog stages.

By Ricardo del Rio

Recent advances in digital signal processing techniques have opened new doors for digital signal processors (DSP).

For the wireless industry, the emphasis is on faster and more accurate analog-to-digital (A/D) converters. Such converters are becoming prominent in the implementation of DSPs in the predominantly analog stages of RF receiver design.

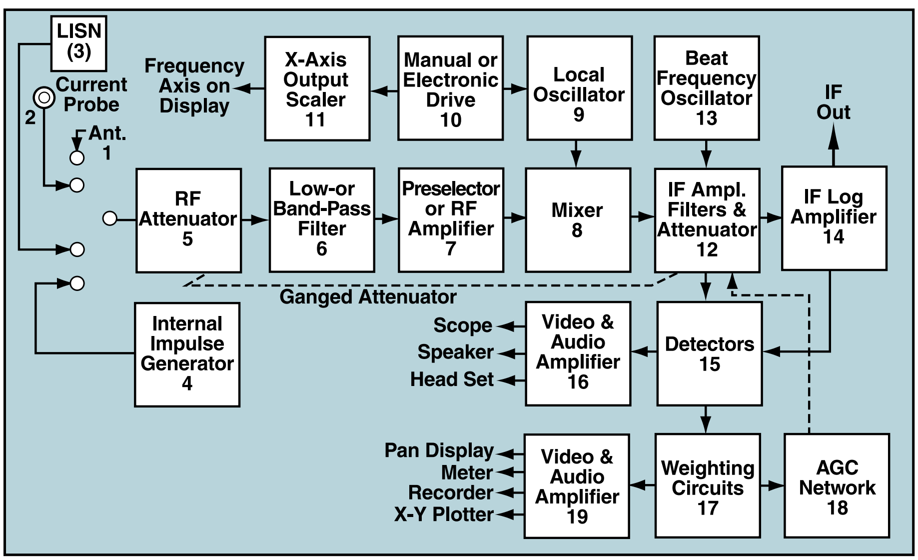

Figure 1. Classic design of an analog receiver.

The basic block diagram of a classic analog receiver design is presented in Figure 1.

Earlier use of the DSP in RF receivers introduced an A/D converter after the amplifier in the last intermediate frequency (IF) stage of superheterodyne receivers. This was done to demodulate and weigh the signal. The initial application was extended as the A/D converters were moved closer to the RF stages. Furthermore, if the A/D converter is relocated after the last mixer of the receiver, it can add to the filtering and signal conditioning.

Digital performance should be introduced as closely as possible to the RF stages. The closer that digital technology is implemented to the RF section, the more advantageous it becomes. Some advantages of early implementation include:

- Improved cost-performance consideration.
- Product enhancement capabilities.
- Reduced alignment and testing requirements.
- Smaller circuit footprint.

The article discusses the undersampling of the signal after the IF analog filter at a frequency that meets Nyquist´s criterion, with respect to its bandwidth rather than its frequency. The technique will be used as an alternative to perform a part of the down conversion task. This technique allows the designer to eliminate one or more analog IF stages and, consequently, move the A/D closer to the RF stages.

The eliminated IF stages can be replaced by a digital circuit consisting of a digital multiplier, a numerically controlled oscillator (NCO) and digital filters, as shown in Figure 2.

The sample periods of the undersampled signal and the signal bandwidth after the IF analog filter are critical parameters of this technique because they impose the design constraints.

The discrete Fourier transform

The undersampling technique is explained in terms of the discrete Fourier transform algorithm.

It is known that a periodic, continuous signal, $x(t)$, can be expressed as a function of its spectral components by means of the Fourier series representation, in which coefficient c_k can be expressed as a function of $x(t)$ in the complex exponential form as:

$$c_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j2\pi f_0 t}dt \tag{1}$$

where f_0 is the fundamental frequency of the signal and c_k, when $k > 0$, is one half of the harmonic amplitude of the signal at the frequency (k x f_0). A complex derivation of the last statement can be found in [3].

The discrete Fourier transform is derived from a finite sequence, $x(n)$, which results from the sampling of a periodic signal, $x_a(t)$, with a sampling period of Ts; that is:

$$c_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j2\pi f_0 t}dt \tag{2}$$

The sampling period is normally chosen of a value such that the fundamental period of the signal, defined as T_0, is a multiple of the former one, that is:

$$T_0 = \frac{1}{f_0} = NT_s \tag{3}$$

where N is the total number of sequence samples.

However, when dealing with the Fourier transform of a discrete sequence instead of a continuous sig-

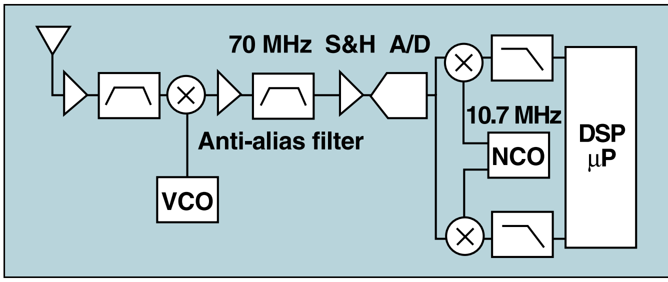

Figure 2. Digital IF stage.

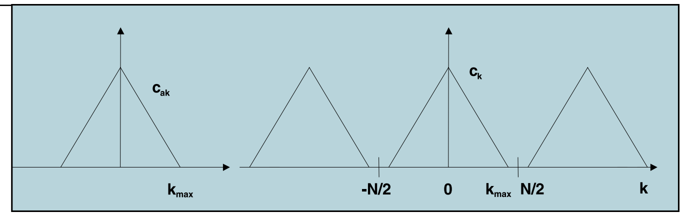

Figure 3. Relation between c_k and c_{ak} components of a band-limited signal.

nal, we must take into account that two additional mathematical transformations take place:

$$t = nT_s \tag{4}$$

and that the integral of expression (1) is converted into a sumatorium. Substituting, then, expressions (2), (3) and (4) into (1), we have:

$$c_k = \frac{1}{NT_s}\sum_{n=0}^{N-1} x_a(nT_s)e^{-j\frac{2\pi}{NT_s}knT_s} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn} \tag{a}$$

from which the discrete Fourier transform, *X(k)*, of the sequence *x(n)* is defined as:

$$X(k) = N \bullet c_k = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn} \tag{b}$$

expression related, as happened with the Fourier series coefficients, to the k harmonic amplitude of the original continuous signal, $x_a(t)$ from which the sequence is derived.

As with the Fourier series, the sequence $x(n)$ can also be expressed as a function of its discrete Fourier transform expression that, to be consistent with the definition of the last one, takes the form:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{-j\frac{2\pi}{N}kn} \tag{c}$$

The aliasing phenomenon

This phenomenon is a direct consequence of the undersampling of a sequence that fails to meet the Nyquist criterion. It also constitutes the basis of the down conversion technique described in this article.

As shown, an analog periodic signal, $x_a(t)$, can be expressed as a Fourier series in its complex-exponential form, that is:

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_{ak}e^{j2\pi k f_0 t} \tag{5}$$

while the sequence resulting from the sampling procedure of the analog signal $x_a(t)$, with a sampling period of Ts, can be expressed as:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{n}kn} = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn} \qquad 6)$$

Now, applying expressions (2) to (5) and taking into account expression (6), it can be observed that the relation existing between both coefficients c_k (related to the definition of the discrete Fourier transform) and c_{ak} (related to the harmonic amplitude of the continous signal $x_a(t)$ is:

$$x_a(nT_s) = \sum_{k=-\infty}^{\infty} c_{ak}e^{j2\pi k f_0 t} = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn} = x(n) \tag{d}$$

Then, taking into account (3):

$$x_a(nT_s) = \sum_{k=-\infty}^{\infty} c_{ak}e^{j\frac{2\pi}{N}kn} = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn} = x(n) \tag{3}$$

Furthermore, (e) can be re-arranged in the following way:

$$x_a(nT_s) = \sum_{k=0}^{N-1}\left(\sum_{r=-\infty}^{\infty} c_{a(k+rN)}\right)e^{j\frac{2\pi}{N}kn} = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn} = x(n) \tag{f}$$

leading to the equality:

$$c_k = \sum_{r=-\infty}^{\infty} c_{a(k+rN)} \tag{7}$$

The c_k sequence is a periodic one in which period is N. This can be seen from:

$$c_{k+N} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}(k+N)n} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}e^{-j2\pi n} = c_k \tag{g}$$

since:

$$e^{-j2\pi n} = 1 \tag{h}$$

The relation between the envelopes of c_k and c_{ak}, for a band-limited signal sampled with a period choosen to meet the Nyquist criterion, is represented graphically in Figure 3, which adds a visual perspective.

Further developing, and assuming $x(n)$ to be a real sequence, we have:

$$c_{-k} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}(-k)n} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn} = c_k^{*} \tag{i}$$

Next, applying both properties at the same time, we get:

$$c_{-k} = c_{N-k} = c_k^{*} \tag{j}$$

Therefore, the number of different c_k coefficients in the discrete Fourier transform of N samples of a sequence is limited to the value of $K_{max} = (N–1)/2$. Consequently, the maximum frequency f_{max}, which can be known from the analog signal with a sampling period of Ts, is:

$$f_{max} = k_{max} \bullet f_0 < \frac{N}{2} f_0 = \frac{N}{2}\frac{1}{NT_s} = \frac{1}{2T_s} \tag{k}$$

This maximum frequency is called the Nyquist frequency. The c_{k+rN} components with a value of frequency $(k+rN)$f higher than that of the Nyquist, will be aliased with those of the c_k. When these components have not been suppressed, by means of filtering for example, we do

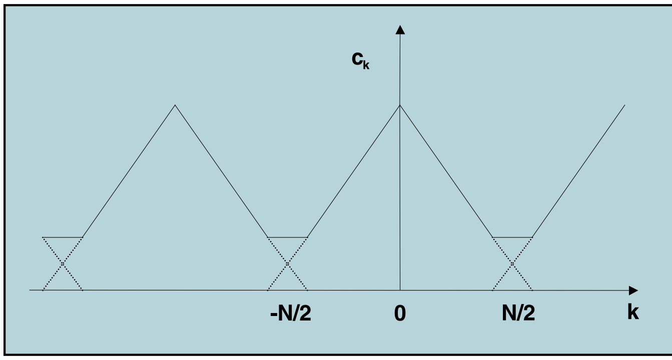

Figure 4. Aliased c_k components of a band-limited signal.

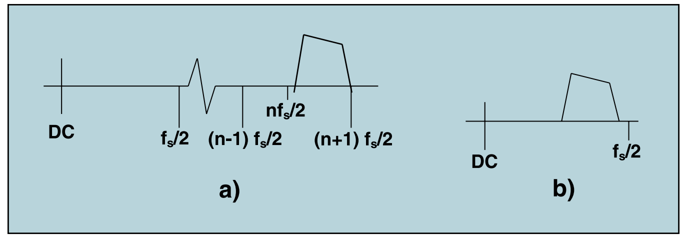

Figure 5. Result of subsampling.

not fulfill the Nyquist criterion. Therefore, an undersampling conditioning appears (the envelope of the discrete Fourier transform components of the band-limited signal of Figure 3, when aliasing exists, is shown in Figure 4).

Application of undersampling

The signal present at the output of the IF filter is an example of a band-limited signal and can be expressed as a function of its spectral content in the form:

$$x_b(t) = \sum_{k=n_1}^{n_2} a_k \cos(2\pi f k - \varphi_k) \tag{8}$$

where f is the maximum common divider of all of the a_k frequencies of the $x_b(t)$.

Actually, the signal, as represented by expression (8), is modified in the mixer stage of a receiver by an analog multiplication with a carrier signal $x_c(t)$ proceeding from the local oscillator. It is expressed by:

$$x_c(t) = a_c \cos(2\pi f_c t + \phi_c) \tag{l}$$

The result of such a multiplication, neglecting the out-of-band components after the IF filter, is a single sidebanded amplitude-modulated signal without carrier suppression, which mathematically can be expressed as:

$$\begin{aligned} & x_c(t) \bullet x_b(t) \\ &= \sum_{k=n_1}^{n_2} \frac{a_c a_k}{2} \cos\left[2\pi(f_c + kf)t + \phi_c - \varphi_k\right] \\ &= \sum_{k=n_1}^{n_2} a_c \bullet c_{bk} \cos\left[2\pi(f_c + kf)t + \phi_c - \varphi_k\right] \end{aligned} \tag{m}$$

where c_{bk} are the Fourier series coefficients of $x_b(t)$. Furthermore, it is possible to make the bandwidth of the signal at the output of the IF filter such that the frequency, f_c of the local oscillator is a multiple of f. The last value will still be the maximum common divider of all frequencies involved in the former expression, simplifying the mathematical analysis.

If the sampling frequency, f_s, is chosen in a way that meets the Nyquist criterion, with respect to its bandwidth rather than its frequency, an undersampling situation appears again. It can be expressed as follows:

$$\begin{aligned} & f_s \geq 2(n_2 - n_1) f \\ & f_s << 2(f_c + n_1 f) \end{aligned} \tag{n}$$

Assuming $f_s = 2(n_2 - n_1)f$, we can perform the discrete Fourier transform $X_b(k)$ over $X_b(t)$, with the former value for the sampling frequency, and end up with a total number of $N = 2(n_2 - n_1)$ samples. Now, taking into account (7), applied to a single sidebanded signal and assuming:

$$n_2 \leq 2(n_2 - n_1) \Rightarrow n_1 \leq \frac{n_2}{2} \tag{o}$$

as a necessary premise to avoid unwanted aliasings, we have:

$$\begin{aligned} & \frac{X_b(n_1)}{2(n_2 - n_1)} = c_{n_1} = c_{b(n_1 + 2r(n_2 - n_1))} \\ & \vdots \\ & \frac{X_b(n_2)}{2(n_2 - n_1)} = c_{n_2} = c_{b(n_2 + 2r(n_2 - n_1))} \end{aligned} \tag{p}$$

where:

$$r = \frac{f_c}{2(n_2 - n_1) f} \tag{q}$$

The result is that we have down converted the coefficients related to the discrete Fourier transform, c_k, to the frequency range $(n_1 f, n_2 f)$. This is with respect to the Fourier series coefficients, c_{bk}, from the frequency range ($f_c + n_1 f$, $f_c + n_2 f$), as was the original premise (Figures 5a and 5b show this down conversion graphically).

Conclusion

Digital techniques are rapidly becoming an integral receiver design criteria. This tendency will continue to increase as digital designs become available that add features and benefits to the both the designer and end user. One example relating to the scheme presented in this article is that the undersample-based down conversion technique allows the A/D converter to operate at a rate consistent with the bandwidths of the signal of interest. This is while the sample and hold (S&H) unit operate with a bandwidth consistent with the IF location of the band of interest. This relaxes the design objective when implementing current A/D converter devices and increases the allowable conversion time for the ADC.

RF

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